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**MAGNETO-OPTICAL STRUCTURE OF THE NICA COLLIDER WITH HIGH CRITICAL ENERGY.**

Methods of increasing the critical energy for the proton option of the NICA collider are investigated. The method of superperiodic modulation of quadrupole gradients is applied. The selection of sextupoles is carried out to suppress the natural chromaticity and compensate for the sextupole component. The Twiss parameters for the proposed structures are given, as well as the dynamic apertures and working points are investigated.

# **PROTON MODE OF NICA COLLIDER.**

First of all the structure of the NICA collider was designed to work in two modes: for experiments with heavy ions and for experiments with polarized protons/deuterons p, d. At the maximum collision energy in the heavy ion collision experiment of 4.5 GeV/nucleon, the critical energy of the magneto-optical structure of the collider is ( In this case, there are no problems with the transition through the critical energy that was originally taken into account when designing. This value of the critical energy was achieved by choosing the frequency of betatron oscillations in the horizontal plane , which is subject to the regularity of the structure of the arches consisting of identical cells of the FODO, must be greater than the maximum value of the Lorentz factor in the entire energy range. This regularity condition of the structure is simultaneously necessary to minimize the modulation of the beam envelope, coinciding with the condition of minimizing the effect of intra-beam scattering in the heavy-ion option with a large particle charge. Small perturbations at the edges of the arch due to the dispersion suppressors by the missing magnet method do not play a fundamental role in all of the above.

Crossing through the critical energy g, a longitudinal instability develops. Threshold current of its instability development proportional to the slip-factor η, which is equal to zero when . At transition, the rate of acceleration of protons using the induction acceleration of the RF1 station is . This rate is too small to avoid the development of instability when the relativistic factor approaches to .

The magnetic rigidity of the rotary magnets is constant . Thus, the maximum possible energy in the proton acceleration experiment is determined , therefore, the gamma-critical must be at the level of , which is obviously higher than the critical energy for an ionic regular structure ( ). In order to exclude the crossing through the critical energy during proton acceleration, a new optical ring structure must be implemented for the proton mode instead of the optical structure of the ion mode. In this optical structure, the critical energy must be higher than the maximum proton energy when the collider is operating for the experiment.

For a proton beam with an intensity of 2×1013 the time of intra-beam heating increases by about 30 times compared to beams of gold ions with an intensity of 6,6×1010. Therefore, the critical energy can rise due to the variation of the dispersion without fear of the influence of intra-beam scattering. Due to the resonant modulation of the dispersion function, the diffusion coefficient for intra-beam scattering increases by 2-3 times, this is not critical both when protons are cooled during accumulation, and when they are grouped at the energy of the experiment.

# **SUPERPERIODIC MODULATION**

The momentum compaction factor is defined as

where – the length of a closed equilibrium orbit, – horizontal dispersion function, – radius of curvature of the equilibrium orbit.

Equation for the dispersion function with biperiodic variable focus

, (2)

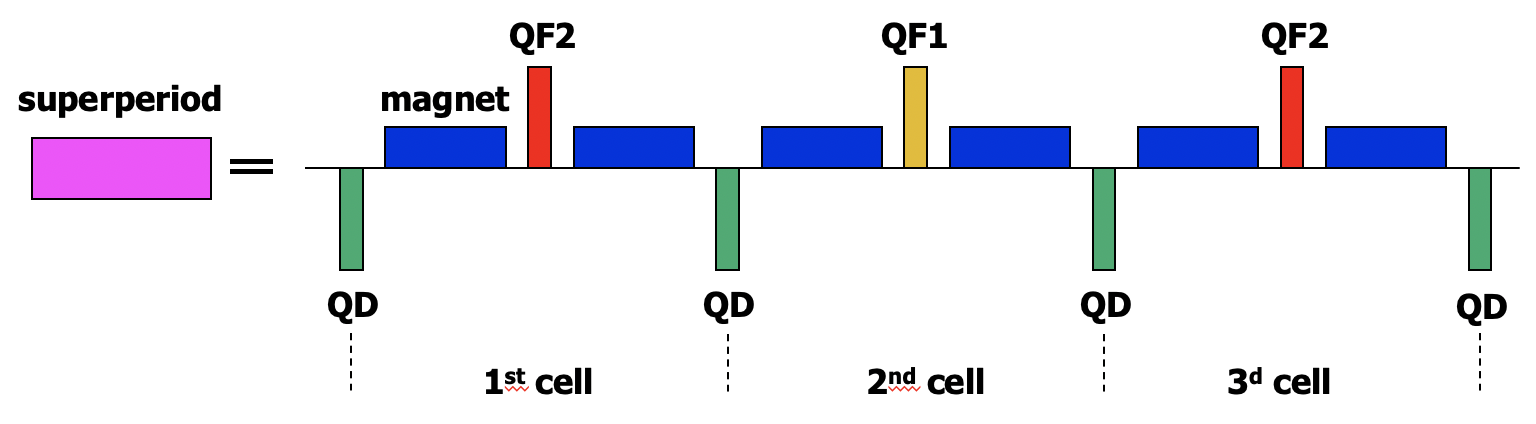
where , ,  *–* gradient of magneto-optical lenses, – superperiodic gradient modulation. A superperiod is defined as a combination of several FODO cells as shown in Figure 1. Thus, in the general case, the coefficient of expansion of the orbit depends on the functions: the curvature of the orbit , gradient and modulation of quadrupole lenses respectively , . In the NICA structure, the regular arrangement of dipole magnets eliminates the possibility of modulating the curvature of the orbit. Therefore, we use only the modulation of the strength of the quadrupole lenses over the length of the superperiod. Function has a periodicity of one period of the focusing cell, has a superperiod periodicity. For one superperiod, the momentum compaction factor is determined by the formula (3), the conclusion of this formula is made in [1]:

Figure 1. Introduction of a superperiod consisting of 3 FODO cells

where – the average value of the curvature, – the number of horizontal betatron oscillations on the length of the arc, S – number of superperiods per arc length, –-th harmonic of the gradient modulation in the Fourier series expansion of the function . Due to the mirror symmetry, the decomposition is performed in cosines. In the absence of superperiodic modulation , formula (3) takes the form , which corresponds to the case of a regular structure. To raise the critical energy, it is necessary to reduce , this means that the expression under the sum sign must be negative, this is realizable under the condition . First harmonic has a dominant influence and for 12 FODO cells, the condition is implemented , , where 3 FODO cells are combined into one superperiod. Thus, due to the tune of betatron oscillations of a multiple of 2π, the arc has the properties of a first-order achromate.

Previously, all formulas were given for the arc, and not for the entire ring of the collider. The insertion of straight sections reduces the degree of modulation of the dispersion function. The average value of the dispersion decreases with a longer orbit length, this means that the momentum compaction factor decreases for the entire accelerator, and the resulting value of the critical energy increases and is determined by the expression:

# **INCREASING THE CRITICAL ENERGY BY SUPERPERIODIC MODULATION.**

To increase the critical energy of the NICA accelerator ring, the possibility of changing the dispersion function by modulating the gradients of the quadrupoles on the rotating arcs of the ring is considered. To do this, consider a superperiod consisting of 3 FODO cells, where the central focusing quadrupole differs from the two edge ones by a larger gradient value.

An important requirement in the design of a magneto-optical structure is to ensure zero dispersion in straight sections to ensure the movement of particles along the equilibrium orbit in these sections. This requirement is easily implemented in the case of creating regular rotating arcs composed of identical superperiods. In this case, by providing a zero dispersion value (as well as the derivative of the dispersion ) at the entrance to the arc, due to the regularity, the output of the arc will also have zero values of the dispersion and its derivative, and therefore on the entire straight section. However, the peculiarity of the given structure of the NICA collider, the presence of missing magnets on the two extreme cells does not make it possible to create a completely regular arc of 4 identical superperiods. Thus, it is necessary to ensure the suppression of dispersion at the edges of the arch. Two possible cases of dispersion suppression are considered:

1. Dispersion suppression with edge superperiods.

To be precise, the dispersion suppression is carried out by using two edge FODO cells located symmetrically on both sides. Figure 2 and 3 above (Edge Suppressor – ES) show a schematic diagram of this magneto-optical structure. As can be seen, the two edge FODO cells differ differ in the presence of a missing-magnet and in these cells, the QFE1 and QFE2 quadrupoles also have different gradients from the main arс quadrupoles and are selected to suppress the dispersion.

1. Dispersion Suppression by the arc, by selecting the gradients of the quadrupoles of the two families.

Figure 2 and 3 below (Arc Suppressor – AS) shows a schematic diagram of this magneto-optical structure. This case differs from the first, all the quadrupoles of the arch belong to the first or second family, and the suppression of dispersion is also provided by only 2 families.

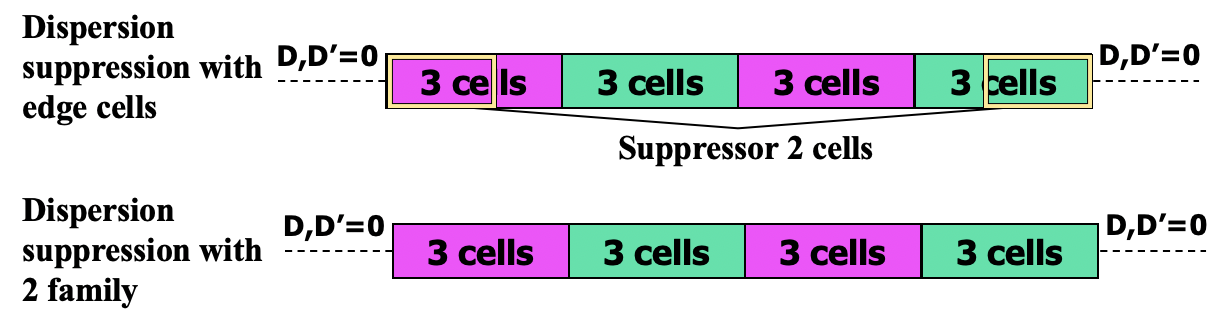
** The defocusing quadrupoles in both the first and second cases belong to only one QD family.

Figure 2. Schematic diagram of two possible dispersion suppression options for the proton option of the NICA collider.

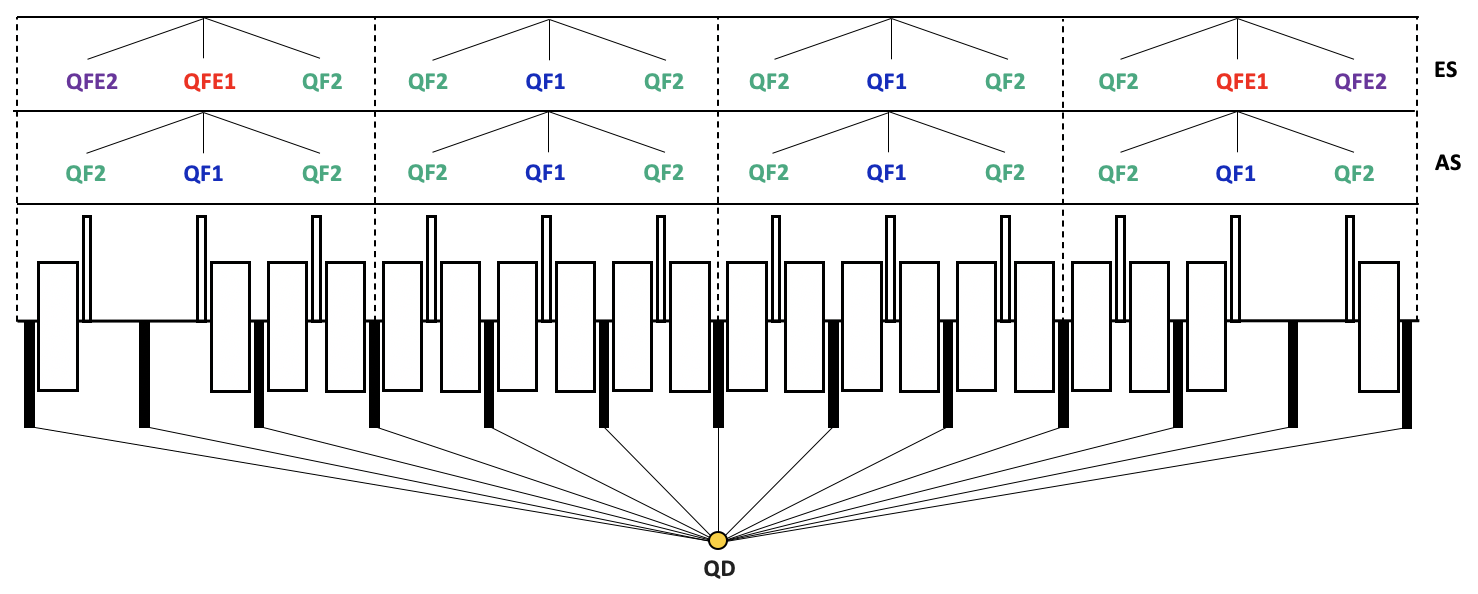


Figure 3. The scheme of arrangement of quadrupoles on the arc of the accelerating ring, providing a variation of the critical energy and performing the function of dispersion suppression in straight sections. The case with the possibility of dispersion suppression by edge superperiods (Edge suppressor) is shown above. Below, the dispersion is suppressed only by matching the gradients of two families of quadrupoles (Arc suppressor).

* 1. **DISPERSION SUPPRESSION BY THE EDGE CELLS OF THE ARC.**
     1. **QUADRUPOLE GRADIENTS.**

The choice of the value of the gradients of the arc quadrupoles is determined by two factors:

1. Obtaining the required value of the critical energy on the entire ring of the collider, which corresponds to ;
2. Provide the number of betatron oscillations on the arc in both planes, thereby satisfying the resonant condition for the number of superperiods .

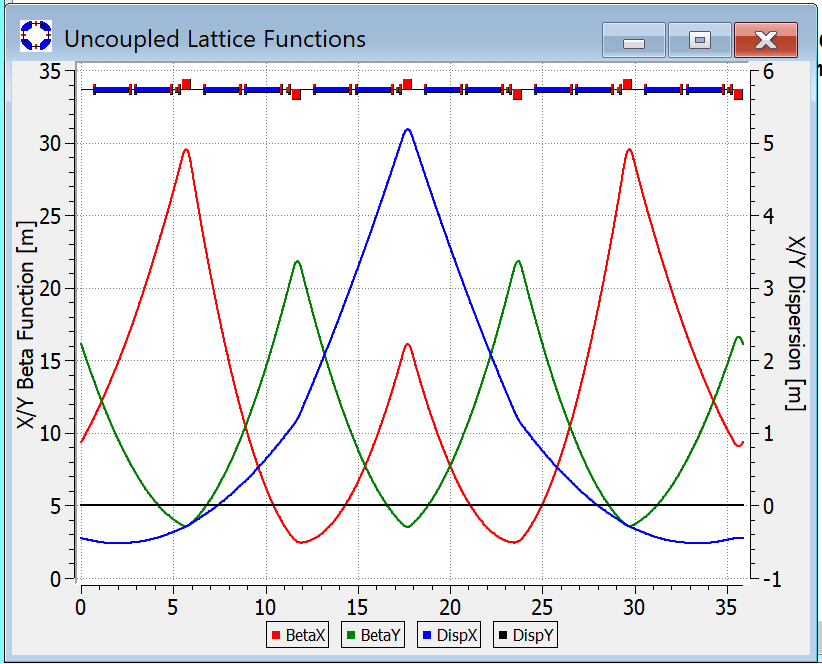
 Based on these conditions, we modulate the superperiod with a phase shift on the superperiod in both planes (Figure 4).

Figure 4. Modulated superperiod

|  |  |
| --- | --- |
| **Quadrupoles** | **Gradient, T/m** |
| QF1 | 28,26481 |
| QF2 | 20,9276 |
| QD | 22,62231 |
| QFE1 | 30,9342 |
| QFE2 | 19,58429 |

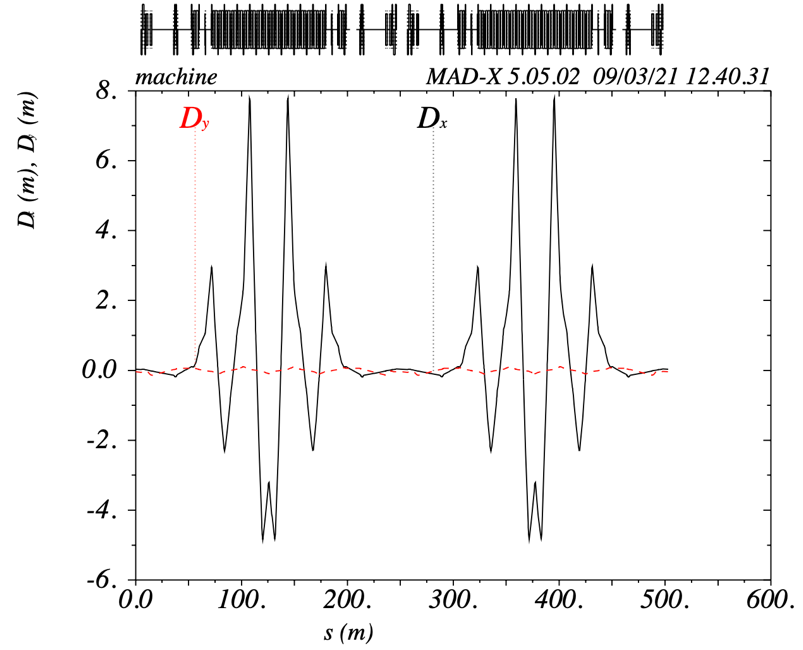
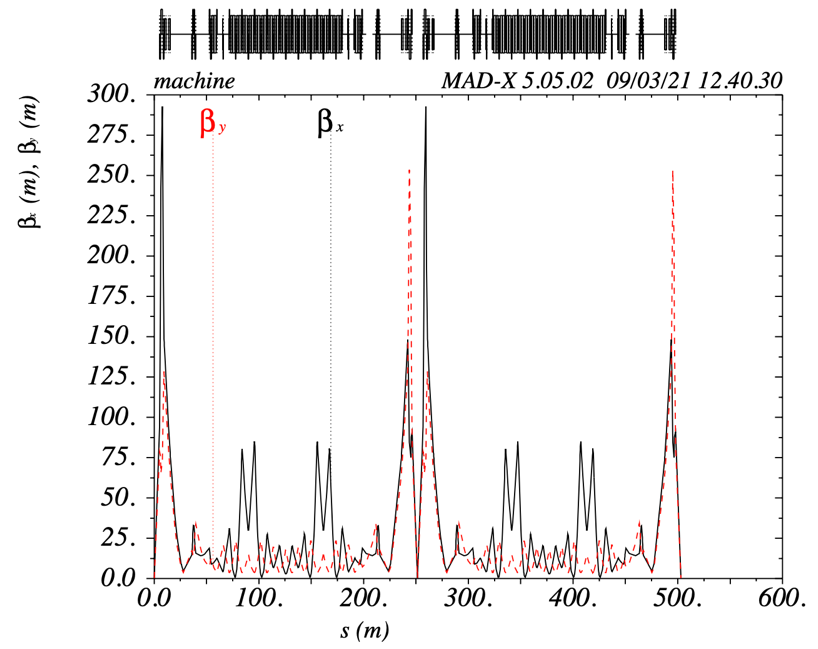


Figure 5. Dispersion suppression by the extreme cells of the arch  
Left – beta functions; right – dispersion functions.

The collider also consists of 2 arcs and 2 straight sections connecting both arcs. In the middle of the straight sections there are collision points, where it is necessary to provide a small value of the beta function to achieve the required luminosity. Figure 6 shows the Twiss parameters of the entire collider ring without introducing the edge quadrupoles QFE1 and QFE2, it is clearly seen that the dispersion is not suppressed in the straight sections. The edge superperiod has a missing magnet in 2 cells, thus making the collider arcs not regular and there is a need to suppress the dispersion in straight sections using the introduction of 2 additional families of QFE1 and QFE2 quadrupoles on the edge of the arc, the beta-function and dispersion function of all entire ring are shown on Figure 5.

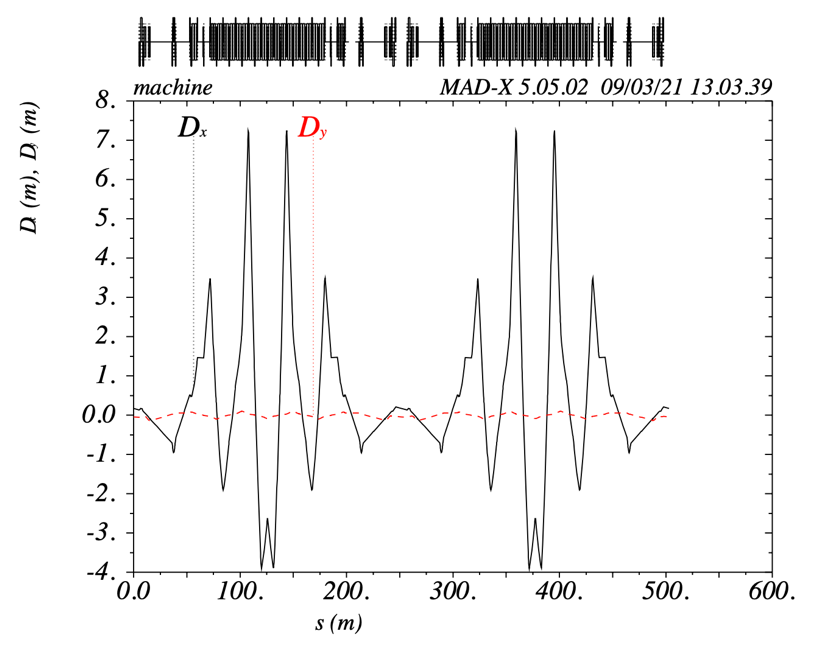
 As a result, the critical energy value is chosen in such a way that , and tune shift over the circle: .

Figure 6. The dispersion function without the dispersion suppressors on the 2 edge cells.

* + 1. **SEXTUPOLES.**

The arrangement of the sextupoles also requires that several parameters are taken into account:

1. First, it is necessary to suppress chromaticity on the entire collider ring;
2. Secondly, to achieve a large value of the dynamic aperture, it is necessary to achieve mutual compensation of the sextupoles and make an achromat of the second orders.

To accomplish the first condition, it is necessary to suppress the natural chromaticity, caused by linear elements-quadrupoles and dipoles. To do this, you need to set the sextupoles in the region of non-zero dispersion – on the arches near the quadrupoles.

Since the tune shift is on the arc . Thus, at each superperiod, a tune shift occurs , including the edge ones. In the described case there is peaks of -function on arc at quadrupoles QF2. Thus, the phase difference between the QF2 quadrupoles of the first and third (second and fourth superperiod) is not a multiple of . Simultaneously, the number of betatron oscillations between the central quadrupoles (QF1 or QFE1) of 1 and 3 or 2 and 4 superperiods . Thus, by placing the sextupoles of the same family next to the central quadrupoles, it will be possible to ensure mutual suppression of the sextupoles. Figure 6 shows a schematic diagram of the arrangement of sextupoles.

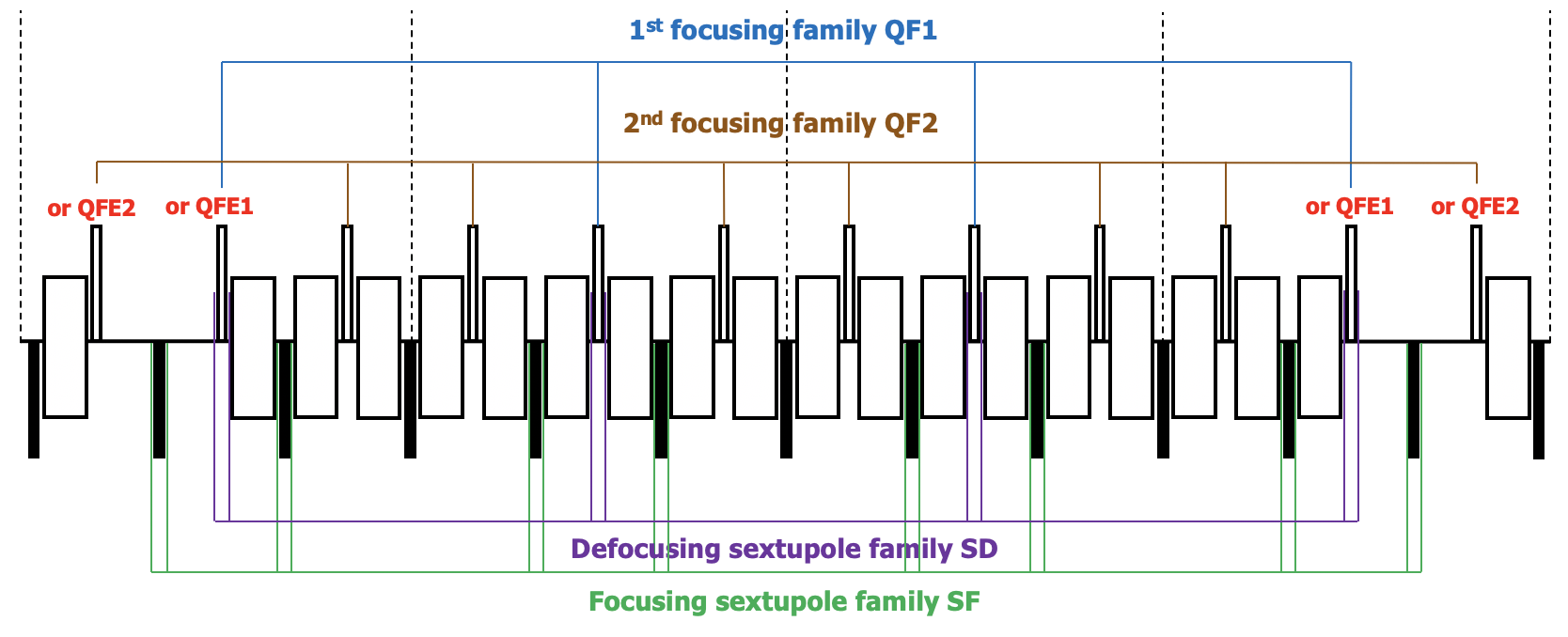
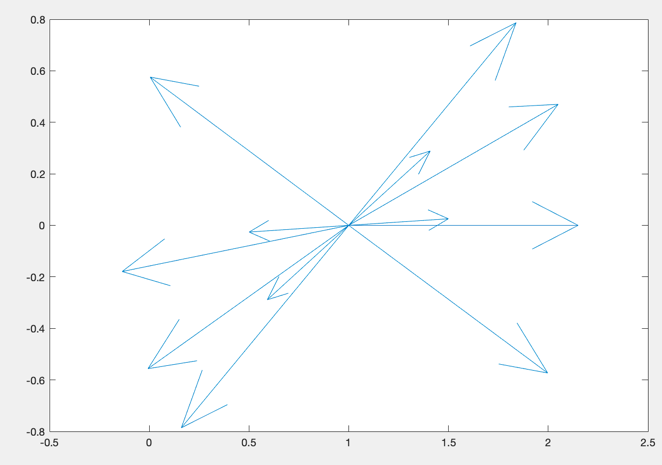
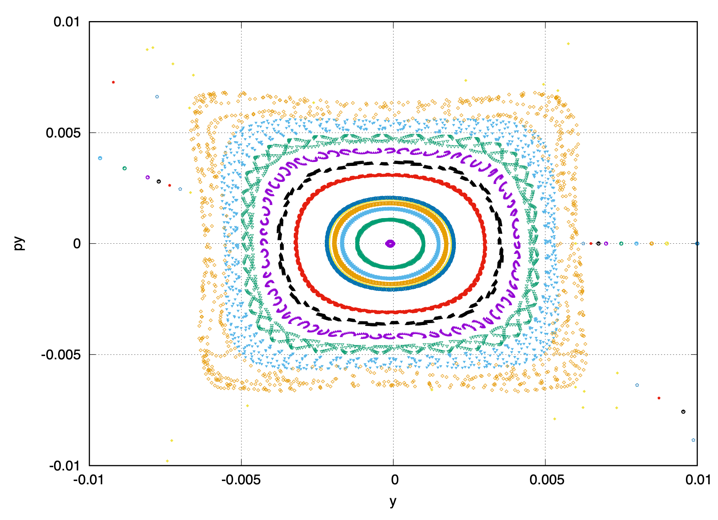
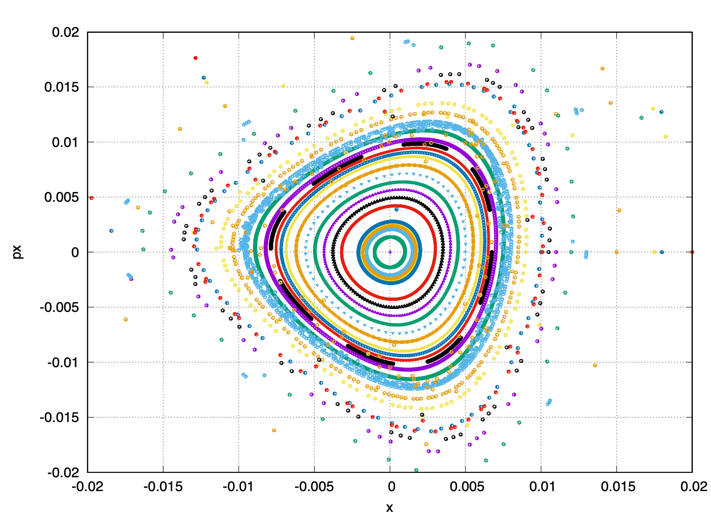
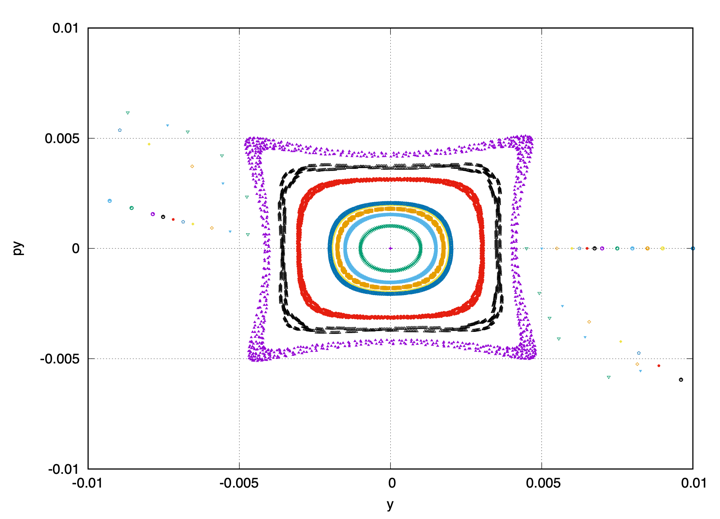
In this case, you need to make sure that the sextupoles are compensated with each other. In Figure 3, the corresponding gradients are represented as vectors, meanwhile, the rotation of the vector reflects the phase transition between the corresponding sextupoles and is equal to the number of betatron oscillations multiplied by (the first sextuple of the arch was taken as a reference point). It can be seen that for each vector there is a vector that is directed in the opposite direction and compensates for it.

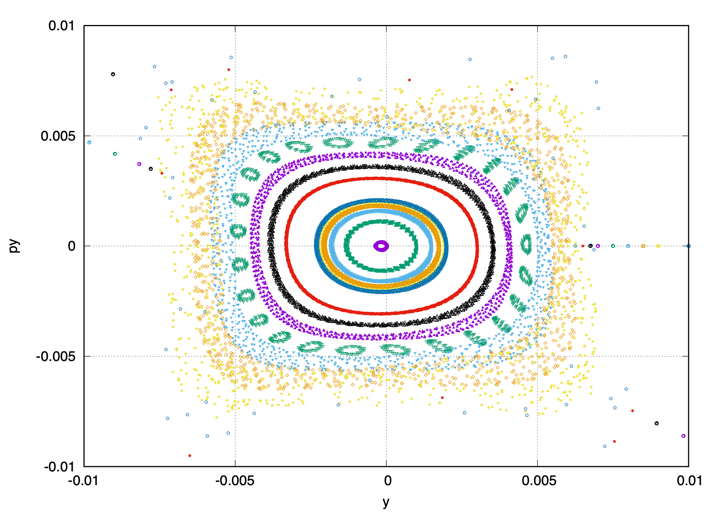
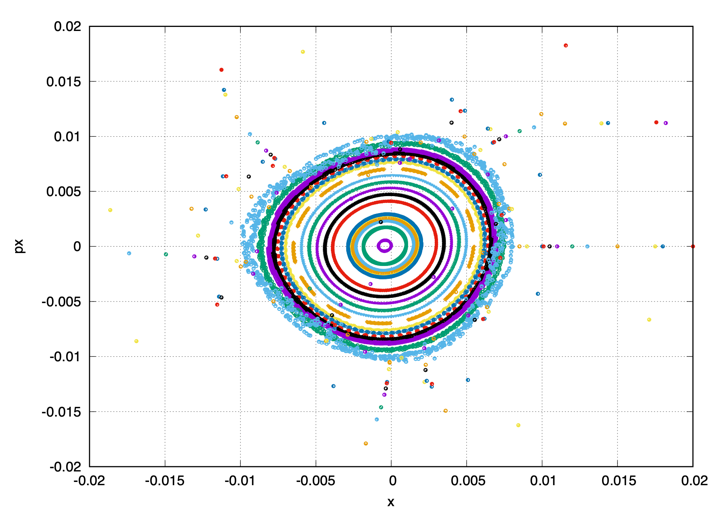
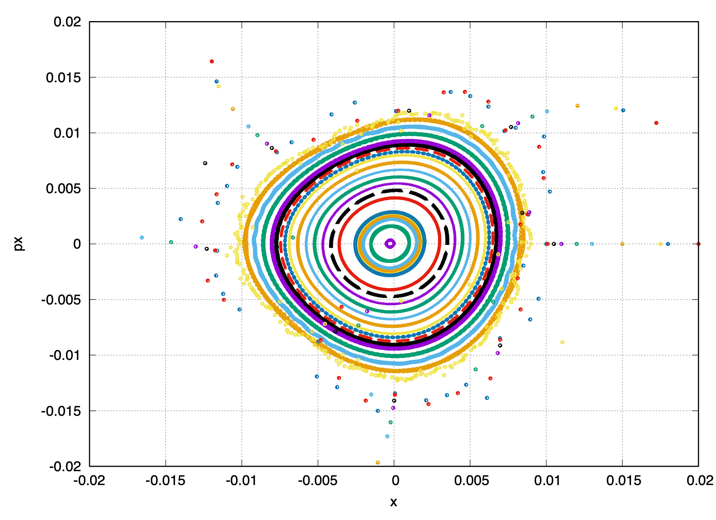
Figure 8. Vectors of sextuple gradients rotated relative to each other taking into account the phase.

Figure 7. Schematic diagram of the arrangement of sextupoles in the case of dispersion suppression by the edge superperiods.

* + 1. **DYNAMIC APERTURE AND OPERATING POINT.**

Working point for the entire ring , the same as for the regular structure. Figure 3 shows the dynamic apertures for this working point in both planes for different . Dynamic aperture in x-plane: ; в y-plane: ;

dp/p = 0.0%

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dp/p = 0.5 %

dp/p = 0.3%

Figure 9. Dynamic aperture for the case dispersion suppression by edge quadrupoles.  
Left – the x-plane; Right – the y-plane.

* 1. **DISPERSION SUPPRESSION BY TWO FAMILIES OF QUADRUPOLES ON THE ARC.**
     1. **QUADRUPOLE GRADIENTS.**

This method shows that it is possible to suppress the dispersion in straight sections using only two families of focusing quadrupoles.

It is important to consider here, as in the first case, to perform:

1. Obtaining the required critical energy value for the entire collider ring, which corresponds to ;
2. Only by using quadrupoles of two families to suppress the variance in straight sections.

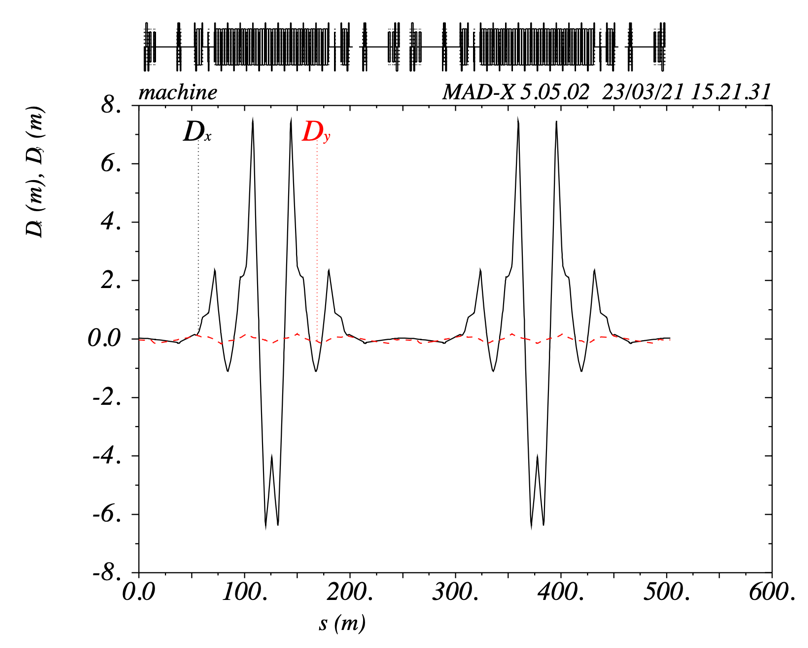
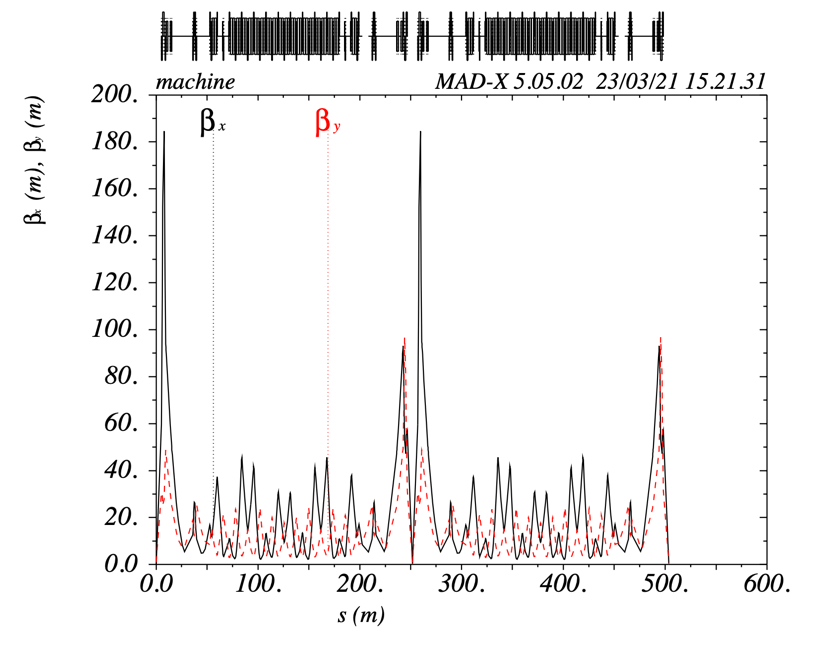


Figure 10. Dispersion suppression by two families of quadrupoles.  
Left – beta-functions; Right – dispersion functions.

Initially, the superperiod is selected, as in the first case with a tune shift on the superperiod . Thus, we get the values of the quadrupoles QF1 and QF2 for the entire arc, including at the edges.

However, it turns out that the dispersion in the straight sections is not suppressed. For suppression, the values of the quadrupole gradients change, but in this case the phase shift on the arc becomes equal to , thus in the x–plane, it is a multiple of .

|  |  |
| --- | --- |
| **Квадруполи** | **Градиент, T/m** |
| QF1 | 28,95 |
| QF2 | 19,91 |
| QD | –22,6 |

In this case, to achieve the required critical energy value, it is necessary to provide a greater modulation of the quadrupole gradients than in the case of dispersion suppression by edge superperiods.

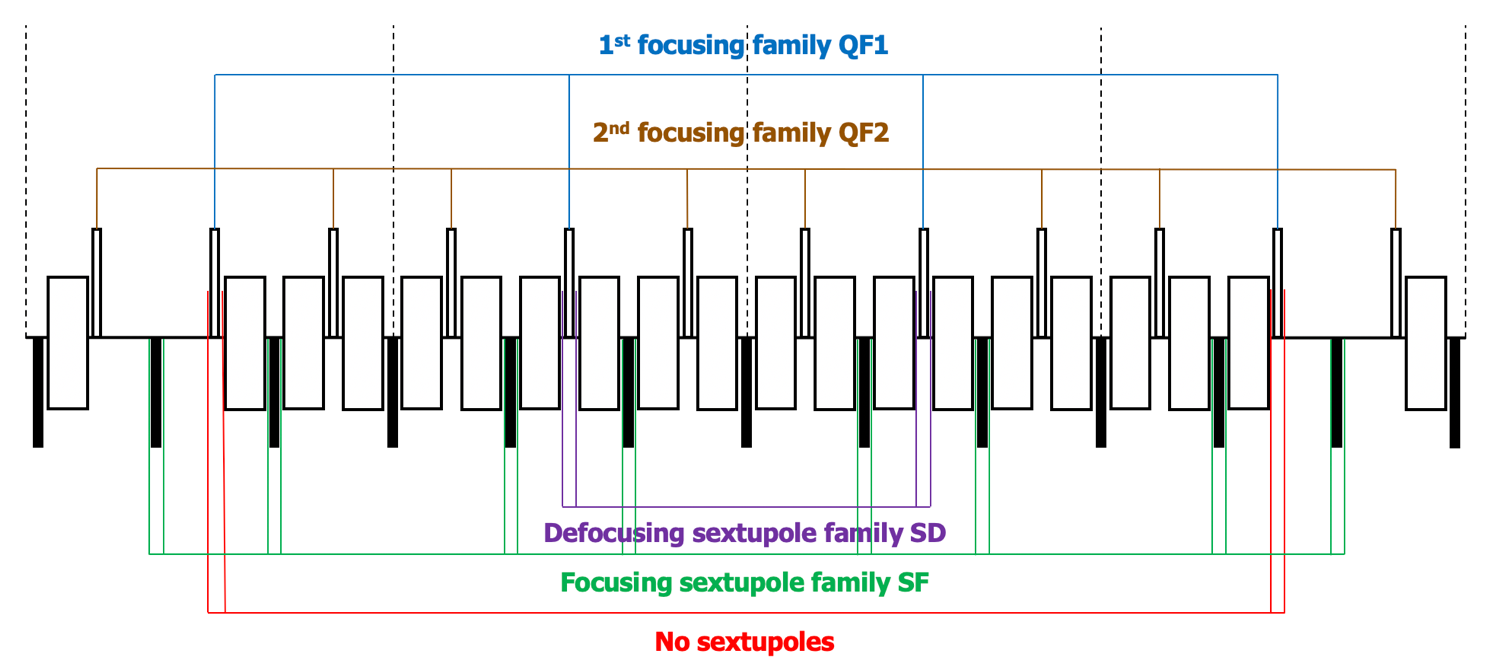
* + 1. **SEXTUPOLES**

Figure 11. Schematic diagram of the arrangement of sextupoles in the case of dispersion suppression by two families of quadrupoles.

Due to the fact, that tune shift on arc not a multiple of , and also between the central quadrupoles is not a multiple , and is equal to , it turns out that the sextupoles do not compensate each other exactly. The arrangement of sextupoles for this case is different from the arrangement of sextupoles in the case of dispersion suppressors at the edges of the arc. The SF family is located next to the central quadrupoles of the superperiod QF1, and SD is located next to the defocusing quadrupoles QD, but only those that surround QF1 on the left and right. However, there are no sextupoles of the focusing family in the edge superperiods. This is done to reduce the influence of sextupoles on the dynamic aperture. The suppression of chromaticity is also possible without them, since the main contribution is made by sextupoles 2 and 3 of the superperiod.

* + 1. **DYNAMIC APERTURE AND OPERATING POINT.**

The working point for the entire ring is , the same as for the regular structure. Figure 3 shows the dynamic aperture for this working point in both planes for different . Dynamic aperture in x-plane: ; в x-плоскости: ;

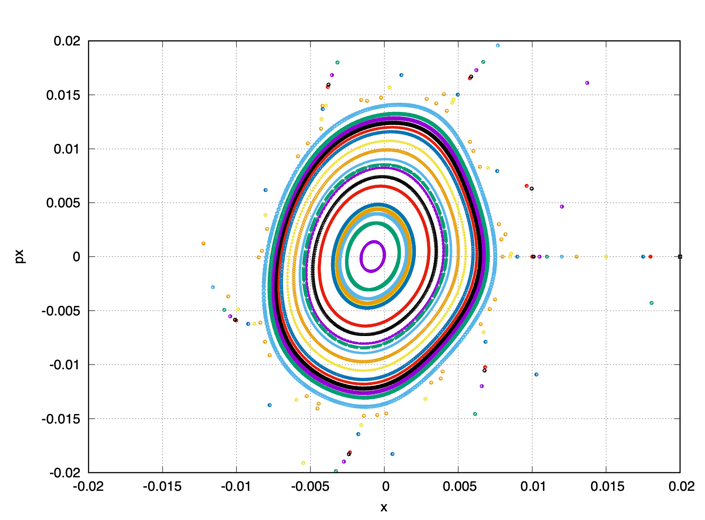
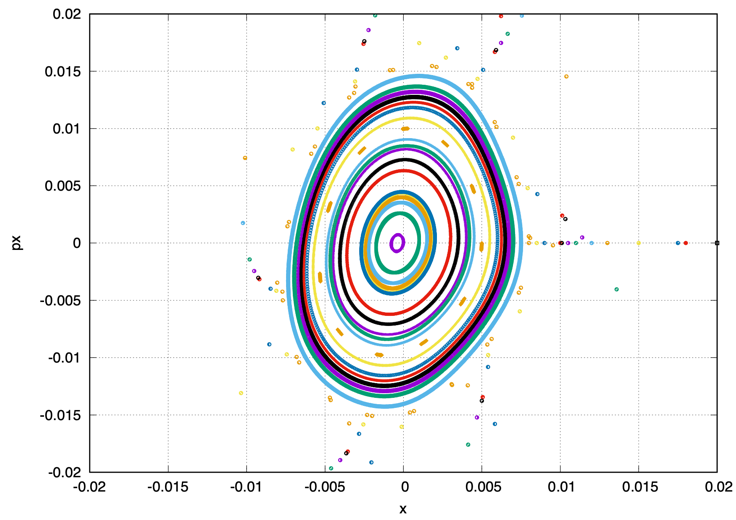
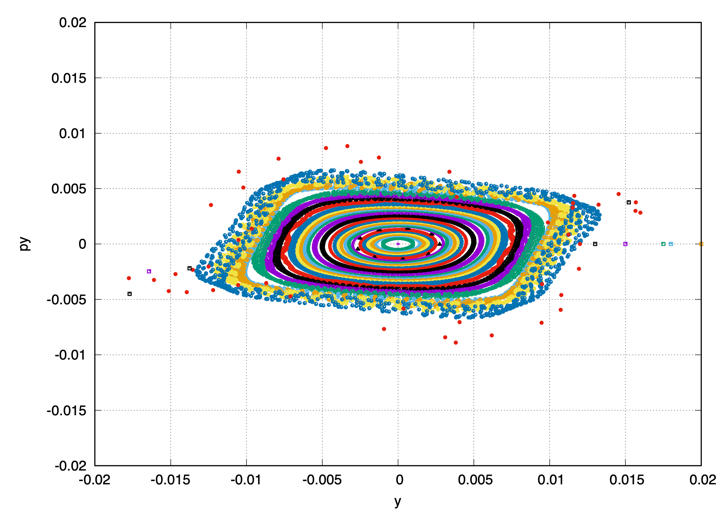
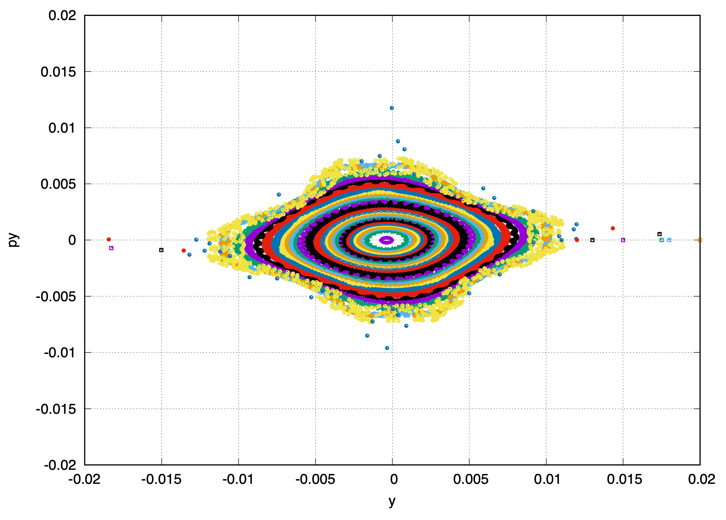
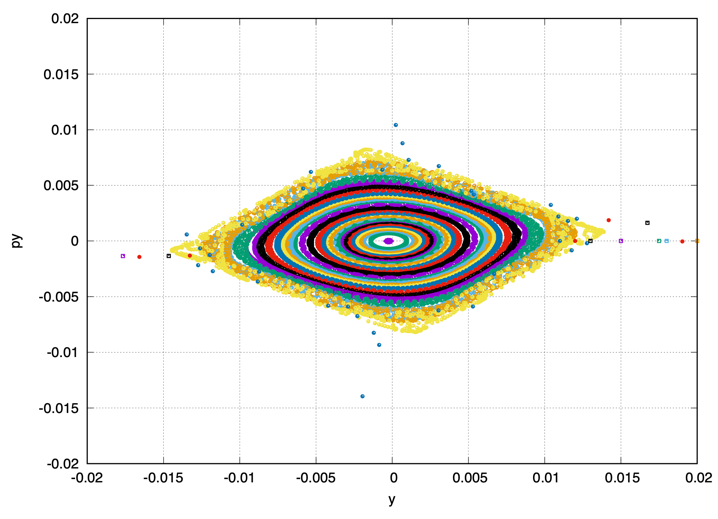
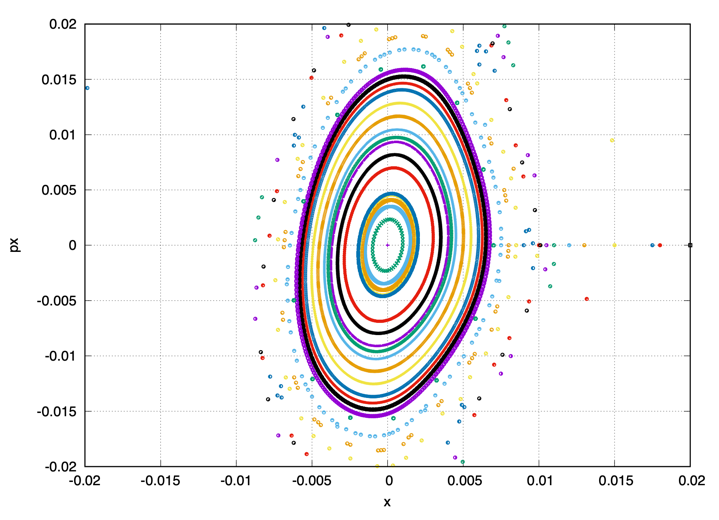
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Figure 12. Dynamic aperture in the case of dispersion suppression by two families of quadrupoles.  
Left – the x-plane; Right – the y-plane.

dp/p = 0.5 %

dp/p = 0.1 %

dp/p = 0.3 %

**LITERATURE.**

1. Yu. V. Senichev and A. N. Chechenin. Theory of “Resonant” Lattices for Synchrotrons with Negative Momentum Compaction Factor. Journal of Experimental and Theoretical Physics, 2007, Vol. 105, No. 5, pp. 988–997
2. Yu. V. Senichev and A. N. Chechenin. Construction of “Resonant” Magneto-Optical Lattices with Controlled Momentum Compaction Factor Journal of Experimental and Theoretical Physics, 2007, Vol. 105, No. 6, pp. 1141–1156.
3. Yu. Senichev, A. Chechenin, S. Kostromin. Variable Transition Energy Lattices based on different periodic cells with various types of dispersion suppressor.
4. Bruno Autin Dispersion Suppression with Missing Magnets in a FODO Structure Application To The CERN Antiproton Accumulator, IEEE Transactions on Nuclear Science Vol. NS-26, No. 3, June 1979.
5. Bernhard J. Holzer. Beam optics and lattice design for particle accelerators. CERN Yellow Report CERN-2013-001, pp.171-206
6. B. Lorentz, A. Lehrach, R. Maier, D. Prasuhn, H. Stockhorst, and R. Tölle, Forschungszentrum Jülich GmbH, D-52425 Jülich, Germany. HESR Linear Lattice Design. Proceedings of EPAC08, Genoa, Italy.
7. P.J.Bryant, Planning Sextupole Families in a Circular Collider. Advanced accelerator physics. Proceedings, 5th Course of the CERN Accelerator School, Rhodos, Greece, September 20-October 1, 1993. Vol. 1, 2.
8. Е. М. Сыресин, А. В. Бутенко, П.Р. Зенкевич, О.С. Козлов, С.Д. Колокольчиков, С.А. Костромин, И.Н. Мешков, Н.В. Митянина, Ю.В. Сеничев, А.О. Сидорин, Г.В.Трубников. Формирование Поляризованных Протонных Пучков в Ускорительном Комплексе Коллайдера NICA.